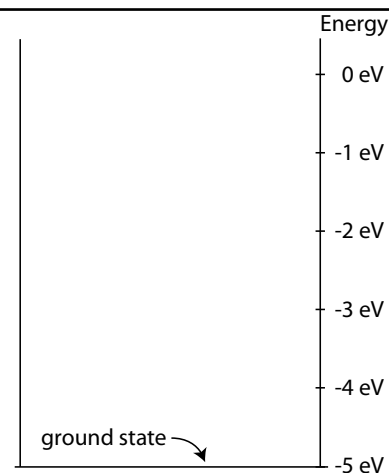


# AP2 Modern Physics

A continuous white-light spectrum illuminates an imaginary monatomic gas with a ground state of  $-5.0$  eV. An optical analyzer reports that wavelengths of  $405$  nm and  $546$  nm are absorbed by the gas.



- (a) Determine the energies of the photons of light absorbed by the gas.
- (b) Show all the excited energy states for the gas atoms in the energy-level diagram above right.
- (c) On the energy-level diagram, indicate all possible electron transitions that would produce bright lines in an emission spectrum by drawing arrows showing the transitions.
- (d) What is the wavelength of the lowest energy photon possibly produced by an electron relaxing from a higher level energy state to a lower level energy state?
- (e) What type of radiation does this wavelength represent?

# AP2 Modern Physics

Answers:

$$(a) E = \frac{hc}{\lambda} = \frac{1240eV \cdot nm}{405nm} = 3.06eV$$

$$E = \frac{hc}{\lambda} = \frac{1240eV \cdot nm}{546nm} = 2.27eV$$

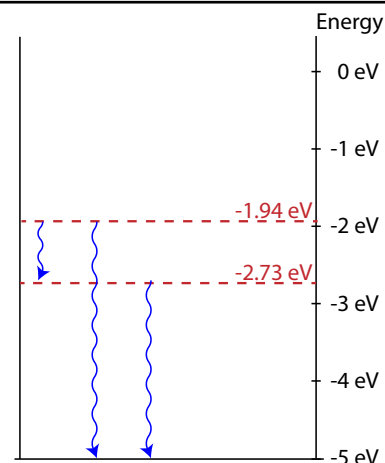
(b) see diagram

(c) see diagram

$$(d) E_{\text{photon}} = E_i - E_f = -1.94eV - -2.73eV = 0.79eV$$

$$E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{1240eV \cdot nm}{0.79eV} = 1570nm$$

(e) Infrared



---

**EK:** 5.B.8 Energy transfer occurs when photons are absorbed or emitted, for example, by atoms or nuclei. 7.C.4 Photon emission and absorption processes are described by probability.

**SP:** 1.1 The student can create representations and models of natural or man-made phenomena and systems in the domain. 1.3 The student can refine representations and models of natural or man-made phenomena and systems in the domain. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

**LO:** 5.B.8.1 The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. 7.C.4.1 The student is able to construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons.

# AP2 Modern Physics

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Which of the following types of nuclear decay particles have the same mass as an electron? (Select two answers.)

- (A) Alpha particle
- (B) Beta particle
- (C) Positron
- (D) Gamma Ray

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**Answer:** (B) beta particle & (C) positron. A beta particle is an electron, and a positron is an anti-electron, a particle with the same mass as an electron but the opposite charge.

**EK:** 1.A.3-c Nuclei have internal structures that determine their properties. There are different types of radioactive emissions from the nucleus. 7.C.3 The spontaneous radioactive decay of an individual nucleus is described by probability.

**SP:** 1.2 The student can describe representations and models of natural or man-made phenomena and systems in the domain.

**LO:** 7.C.3.1 The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay.

# AP2 Modern Physics

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A specific radioactive material is found to decay such that 25% of the original sample remains after 100 years. What is the half-life of the material?

- (A) 25 years
- (B) 33.3 years
- (C) 50 years
- (D) 66.7 years



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**Answer:** (C) 50 years. After one half-life (50 years), half the sample remains. After a second half life (another 50 years), one quarter (25%) of the sample remains.

**EK:** 7.C.3 The spontaneous radioactive decay of an individual nucleus is described by probability.

- a. In radioactive decay processes, we cannot predict when any one nucleus will undergo a change; we can only predict what happens on the average to a large number of identical nuclei.
- c. The time for half of a given number of radioactive nuclei to decay is called the half-life.
- d. Different unstable elements and isotopes have vastly different half-lives, ranging from small fractions of a second to billions of years.

**SP:** 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

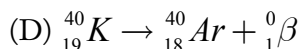
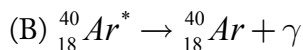
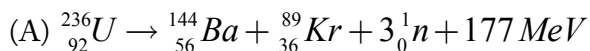
**LO:** 7.C.3.1 The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay.

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# AP2 Modern Physics

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For each of the following reaction equations, identify the type of nuclear reaction.



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**Answers:**

- (A) fission
- (B) gamma decay
- (C) fusion
- (D)  $\beta^+$  decay

**EK:** 1.A.3-c Nuclei have internal structures that determine their properties. There are different types of radioactive emissions from the nucleus. 7.C.3 The spontaneous radioactive decay of an individual nucleus is described by probability.

**SP:** 1.2 The student can describe representations and models of natural or man-made phenomena and systems in the domain.

**LO:** 7.C.3.1 The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay.

# AP2 Modern Physics

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Which of the following statements concerning Einstein's Theory of Special Relativity is true?

- (A) Events occur simultaneously for all observers in all reference frames.
- (B) The speed of light has the same value for all observers in all reference frames.
- (C) A clock moving at a high speed runs slower than a clock at rest.
- (D) A car moving at a high speed is longer than a car at rest.

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**Answer:** (B) The speed of light has the same value for all observers in all reference frames. This is the foundation of special relativity.

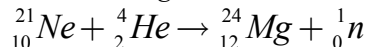
**EK:** 1.D.3 Properties of space and time cannot always be treated as absolute. Measurements of length and time depend on speed. (Qualitative treatment only.)

**SP:** 1.2 The student can describe representations and models of natural or man-made phenomena and systems in the domain. 7.1 The student can connect phenomena and models across spatial and temporal scales.

**LO:** 1.D.3.1 The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can "disagree" about some time and distance intervals.]

# AP2 Modern Physics

The following fusion reaction occurs in stars:



Given the masses listed in the table at right, how much energy is released in the reaction?

Particle	Mass (u)
${}_0^1n$	1.007825
${}_2^4\text{He}$	4.002603
${}_{10}^{21}\text{Ne}$	20.993849
${}_{12}^{24}\text{Mg}$	23.985042

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**Answer:** 3.34 MeV. Add up the mass of all the reagents, add up the mass of all the products, and the difference is the amount of mass converted into energy. 1 universal mass unit (u) results in 931 MeV.

**EK:** 1.C.4 In certain processes, mass can be converted to energy and energy can be converted to mass according to  $E=mc^2$ , the equation derived from the theory of special relativity. 4.C.4 Mass can be converted into energy and energy can be converted into mass. 5.B.11 Beyond the classical approximation, mass is actually part of the internal energy of an object or system with  $E = mc^2$ .

**SP:** 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

**LO:** 1.C.4.1 The student is able to articulate the reasons that the theory of conservation of mass was replaced by the theory of conservation of mass-energy. 4.C.4.1 The student is able to apply mathematical routines to describe the relationship between mass and energy and apply this concept across domains of scale. 5.B.11.1 The student is able to apply conservation of mass and conservation of energy concepts to a natural phenomenon and use the equation  $E = mc^2$  to make a related calculation.

# AP2 Modern Physics

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Which of the following best supports the particle nature of light?

- (A) Compton Effect
- (B) Diffraction
- (C) Doppler Effect
- (D) Interference

---

**Answer:** (A) The Compton Effect showed that photons have momentum and they obey the laws of conservation of energy and conservation of momentum.

**EK:** 1.D.2 Certain phenomena classically thought of as waves can exhibit properties of particles. 6.F.4 The nature of light requires that different models of light are most appropriate at different scales.

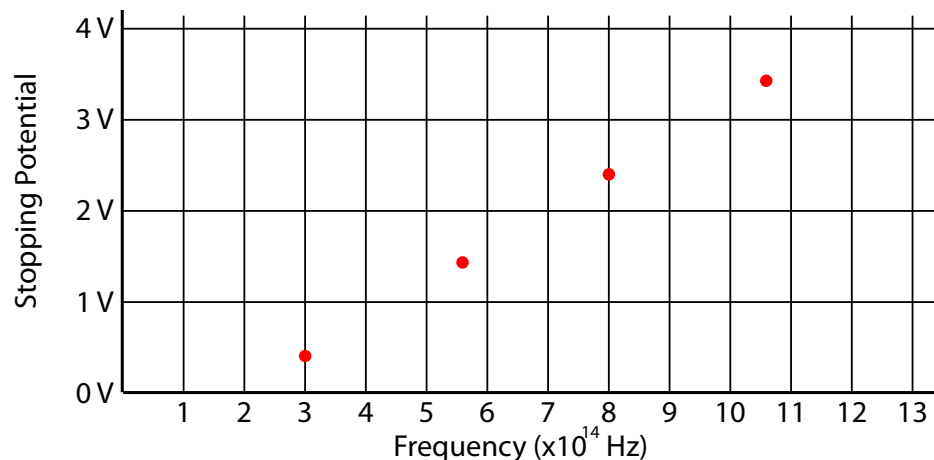
**SP:** 1.5 The student can reexpress key elements of natural phenomena across multiple representations in the domain. 6.3 The student can articulate the reasons that scientific explanations and theories are refined or replaced.

**LO:** 6.F.4.1 The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter.



# AP2 Modern Physics

An unknown metal is irradiated with monochromatic light in an experiment. As the frequency of the light is increased, electrons are ejected and collected in a photoelectric circuit. A reverse-bias potential is applied to the circuit at each specific frequency until the current just reaches zero. This stopping potential is recorded for the various incident frequencies and plotted below.



- (a) What is the maximum speed an emitted photoelectron could obtain if this metal were irradiated with light of wavelength 429 nm?
- (b) Determine the cutoff frequency of the metal.
- (c) Determine the work function of the metal.
- (d) Use the graph to calculate Planck's Constant. Compare to the accepted value and calculate a percent error.
- (e) Draw a line representing a different metal that has a work function of 1.5 volts.

# AP2 Modern Physics

## Answers:

- (a) First find the energy of the photoelectron. To do this, determine its frequency and use the graph to find the stopping potential of the emitted particle:

$$v = f\lambda \rightarrow f = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{429 \times 10^{-9} \text{ m}} = 7 \times 10^{14} \text{ Hz}$$

This stopping potential correlates to the energy of the emitted photoelectron. Use this energy to find the speed of the emitted electron.

$$2eV = 3.2 \times 10^{-19} \text{ J}$$

$$KE = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(3.2 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 838,000 \text{ m/s}$$

- (b)  $2 \times 10^{14}$  Hz right from the graph.  
 (c) You can extend the graph to find the y-intercept, which is the work function, or use the equation of the line:

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4\text{V} - 0\text{V}}{12 \times 10^{14} \text{ Hz} - 2 \times 10^{14} \text{ Hz}} = 4 \times 10^{-15} \text{ V} \cdot \text{s}$$

Next utilize the formula for a line,  $y=mx+b$ , to solve for the y-intercept.

$$y = mx + b \rightarrow b = y - mx \xrightarrow{\substack{y=2\text{V}, x=7 \times 10^{14} \text{ Hz} \\ m=4 \times 10^{-15} \text{ V} \cdot \text{s}}} b = 2\text{V} - (4 \times 10^{-15} \text{ V} \cdot \text{s})(7 \times 10^{14} \text{ Hz}) = -0.8\text{V}$$

The work function is therefore 0.8V.

- (d) The slope leads you to Planck's Constant, though you must treat the y-axis as the electron energy by multiplying the potential by the charge on an electron.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4\text{eV} \times 1.6 \times 10^{-19} \text{ J/eV}}{10 \times 10^{14} \text{ Hz}} = 6.4 \times 10^{-34} \text{ J} \cdot \text{s}$$

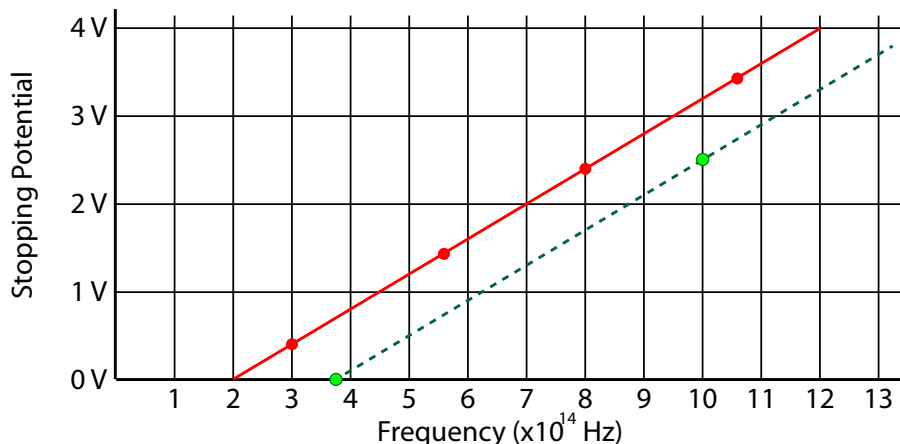
Next find the error in your measurement:

$$\% \text{err} = \frac{|h_{\text{exp}} - h_{\text{acc}}|}{h_{\text{acc}}} \times 100\% \rightarrow$$

$$\% \text{err} = \frac{|6.4 \times 10^{-34} \text{ J} \cdot \text{s} - 6.63 \times 10^{-34} \text{ J} \cdot \text{s}|}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \times 100\%$$

$$\% \text{err} = 3.5\%$$

- (e) This line will have the same slope as the previous line, but the y-intercept will be -1.5, so you can find two points on the line using the equation of the line  $y=mx+b$ , which in this case will be  $y=(4 \times 10^{-15} \text{ V} \cdot \text{s})x - 1.5\text{V}$ . Example points could include  $(3.75 \times 10^{14} \text{ Hz}, 0)$  and  $(10 \times 10^{14} \text{ Hz}, 2.5\text{V})$ . Connect the points to get the line!



# AP2 Modern Physics

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**EK:** 6.F.3 b & c Photons are individual energy packets of electromagnetic waves, with  $E_{\text{photon}} = hf$ , where  $h$  is Planck's constant and  $f$  is the frequency of the associated light wave. b) For the short-wavelength portion of the electromagnetic spectrum, the energy per photon can be observed by direct measurement when electron emissions from matter result from the absorption of radiant energy. c) Evidence for discrete energy packets is provided by a frequency threshold for electron emission. Above the threshold, emission increases with the frequency and not the intensity of absorbed radiation. The photoelectric effect should be included as an example.

**SP:** 1.1 The student can create representations and models of natural or man-made phenomena and systems in the domain. 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 5.1 The student can analyze data to identify patterns or relationships. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

**LO:** 6.F.3.1 The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect.

# AP2 Modern Physics

An X-ray photon with frequency  $f$  undergoes an elastic collision with a stationary electron (mass  $m_e$ ) and is scattered. The frequency of the scattered X-ray photon is  $f'$ .

(a) Determine the kinetic energy of the electron after the collision in terms of  $f$ ,  $f'$ , and fundamental constants.

(b) Determine the magnitude of the momentum of the scattered electron in terms of  $f$ ,  $f'$ ,  $m_e$ , and fundamental constants.

(c) Determine the de Broglie wavelength of the electron in terms of  $f$ ,  $f'$ ,  $m_e$ , and fundamental constants.

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## Answers:

(a) Utilizing conservation of energy:  $hf = K_{electron} + hf' \rightarrow K_{electron} = hf - hf' = h(f - f')$

(b) First find the speed of the electron:  $K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2h(f - f')}{m_e}}$

Next solve for the momentum:  $p = mv = m_e \sqrt{\frac{2h(f - f')}{m_e}} = \sqrt{2m_e h(f - f')}$

(c)  $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e h(f - f')}} = \sqrt{\frac{h}{2m_e(f - f')}}$

**EK:** 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after. 7.C.2b The allowed states for an electron in an atom can be calculated from the wave model of an electron. b) The de Broglie wavelength of an electron can be calculated from its momentum, and a wave representation can be used to model discrete transitions between energy states as transitions between standing waves.

**SP:** 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

**LO:** 5.D.1.6 The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. 7.C.2.1 The student is able to use a standing wave model in which an electron orbit circumference is an integer multiple of the de Broglie wavelength to give a qualitative explanation that accounts for the existence of specific allowed energy states of an electron in an atom.

# AP2 Modern Physics

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An alien space ship attacks a planet by shooting a laser beam at the planet every five seconds. If the ship is approaching the planet at high speed, which of the following is true for an observer situated on the planet?

- (A) The laser beams strike the planet at a frequency of 0.2 Hz.
- (B) The laser beams strike the planet at a frequency less than 0.2 Hz.
- (C) The laser beams strike the planet at a frequency greater than 0.2 Hz.
- (D) More information is needed.



---

**Answer:** (C) The laser beams strike the planet at a frequency greater than 0.2 Hz. As the spaceship approaches the planet while emitting laser beams at a frequency of 0.2 Hz, the observer on the planet will observe a frequency shift to a higher frequency consistent with the Doppler Effect.

**EK:** 1.D.3 Properties of space and time cannot always be treated as absolute. 6.B.5 The observed frequency of a wave depends on the relative motion of source and observer. This is a qualitative treatment only.

**SP:** 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

**LO:** 1.D.3.1 The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. 6.B.5.1 The student is able to create or use a wave front diagram to demonstrate or interpret qualitatively the observed frequency of a wave, dependent upon relative motions of source and observer.

# AP2 Modern Physics

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Barry and Dawn meet at 11:59 pm every New Year's Eve at the top of the Sears Tower in Chicago. This year, Barry, a pilot, travels 1,500,000 kilometers during the year, while Dawn, a computer programmer, travels 20,000 km during the year. For the period between 11:59 pm last year and 11:59 pm this year, which of the following is true?

- (A) Barry has aged less than Dawn.
- (B) Dawn has aged less than Barry.
- (C) They have both aged the same amount.
- (D) Not enough information is given.



---

**Answer:** (A) Barry has aged less than Dawn. According to relativity theory, the faster an object moves, the less time it experiences. Put another way, the two events at 11:59 pm are separated by a specific amount of space-time. Since Barry travels further in space, he must experience less time than Dawn, who travels less in space, and therefore experiences more time.

**EK:** 1.D.3 Properties of space and time cannot always be treated as absolute. Measurements of length and time depend on speed. (Qualitative treatment only.)

**SP:** 1.2 The student can describe representations and models of natural or man-made phenomena and systems in the domain. 7.1 The student can connect phenomena and models across spatial and temporal scales.

**LO:** 1.D.3.1 The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can “disagree” about some time and distance intervals.]

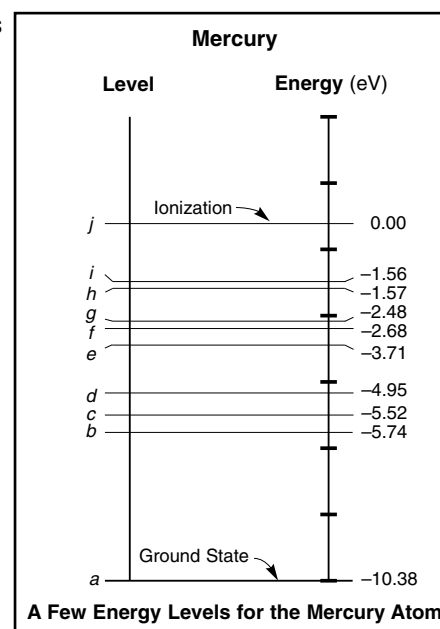
# AP2 Modern Physics

An experiment is set up such that a beam of monochromatic light passes through a diffraction grating with 3000 lines/cm, creating a diffraction pattern on a screen located 0.1 meter from the grating. The distance between the central maximum and the next nearest bright line on the screen is 1.32 cm.

- (a) Determine the wavelength of the incident light.
- (b) What is the energy of each incident photon (in eV)?

The incident light is created by a filtered mercury arc lamp. A few energy levels of mercury are shown at right.

- (c) Which electron transition in the lamp is most likely responsible for creation of the photons striking the diffraction grating?



- (d) The diffraction grating is replaced with a new grating containing 2000 lines/cm. Does the distance between maxima
- increase
  - decrease
  - remain the same

Explain your reasoning.

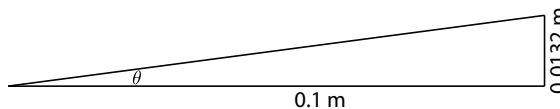
# AP2 Modern Physics

## Answers:

(a) First find the spacing between slits:  $\frac{.01m}{3000lines} = 3.33 \times 10^{-6} m$

Next determine a value for  $\sin \theta$  given your geometric setup:

$$\theta = \tan^{-1} \left( \frac{0.0132m}{0.1m} \right) = 7.52^\circ \rightarrow \sin \theta = \sin(7.52^\circ) = 0.1308$$



Then solve for the wavelength of light using the diffraction grating equation:

$$d \sin \theta = m\lambda \rightarrow \lambda = \frac{d \sin \theta}{m} = \frac{3.33 \times 10^{-6} m \cdot 0.1308}{1} = 4.36 \times 10^{-7} m = 436 nm$$

(b)  $E = hf = \frac{hc}{\lambda} = \frac{1240 eV \cdot nm}{436 nm} = 2.84 eV$

(c) Level f to level c:  $E_{\text{photon}} = E_i - E_f = -2.68 eV - -5.52 eV = 2.84 eV$

(d) Decrease. With fewer slits per cm, the width of the slits increases. Therefore, the angle must decrease.

**EK:** 1.A.4 Atoms have internal structures that determine their properties. 5.B.8 Energy transfer occurs when photons are absorbed or emitted, for example, by atoms or nuclei. 6.C.3 When waves pass through a set of openings whose spacing is comparable to the wavelength, an interference pattern can be observed. Examples should include monochromatic double-slit interference. 6.F.3 Photons are individual energy packets of electromagnetic waves, with  $E_{\text{photon}} = hf$ , where  $h$  is Planck's constant and  $f$  is the frequency of the associated light wave. 6.F.4 The nature of light requires that different models of light are most appropriate at different scales. 7.C.4 Photon emission and absorption processes are described by probability.

**SP:** 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 1.5 The student can reexpress key elements of natural phenomena across multiple representations in the domain. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

**LO:** 1.A.4.1 The student is able to construct representations of the energy-level structure of an electron in an atom and to relate this to the properties and scales of the systems being investigated. 5.B.8.1 The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. 6.C.3.1 The student is able to qualitatively apply the wave model to quantities that describe the generation of interference patterns to make predictions about interference patterns that form when waves pass through a set of openings whose spacing and widths are small compared to the wavelength of the waves. 6.F.4.1 The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter. 7.C.4.1 The student is able to construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons.



# AP2 Modern Physics

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The de Broglie wavelength of a particle that has a kinetic energy,  $K$ , is  $\lambda$ . The wavelength  $\lambda$  is proportional to

- (A)  $K^2$
- (B)  $1/K$
- (C)  $1/\sqrt{K}$
- (D)  $\sqrt{K}$

---

**Answer:** (C) Wavelength is proportional to  $1/p$  and  $p^2$  is proportional to  $K$ , therefore wavelength is proportional to  $1/\sqrt{K}$ .

**EK:** 6.G.2 Under certain regimes of energy or distance, matter can be modeled as a wave. The behavior in these regimes is described by quantum mechanics.

**SP:** 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

**LO:** 6.G.2.1 The student is able to articulate the evidence supporting the claim that a wave model of matter is appropriate to explain the diffraction of matter interacting with a crystal, given conditions where a particle of matter has momentum corresponding to a de Broglie wavelength smaller than the separation between adjacent atoms in the crystal.

# AP2 Modern Physics

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A stationary nucleus of uranium-232 decays into thorium-228 by means of alpha decay. The masses of each are given below.

$$m_{\text{U}232} = 232.0317 \text{ amu}$$

$$m_{\text{Th}228} = 228.0287 \text{ amu}$$

$$m_{\text{alpha}} = 4.0026 \text{ amu}$$

(a) Calculate the energy released in this decay in MeV.

(b) Show that the ratio of kinetic energy of the alpha particle to the kinetic energy of the thorium daughter nucleus is approximately 57:1.

(c) Give an explanation of why the uranium-232 nucleus decays by emitting an alpha particle instead of 2 individual neutrons and 2 individual protons.

# AP2 Modern Physics

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## Answers:

- (a)  $\Delta E = \Delta m \times 931 \text{ MeV}/u = 0.0004 \times 931 \text{ MeV}/u = 0.37 \text{ MeV}$
- (b) The momentum of each daughter nucleus must be equal in order to satisfy conservation of linear momentum. Since  $K = p^2/2m$ ,  $K_{\text{daughter}}$  must be proportional to  $1/m$ , therefore  $K_{\alpha}/K_{\text{Th}} = m_{\text{Th}}/m_{\alpha} = 228/4 = 57:1$
- (c) The sum of the masses of the four individual nucleons is greater than one alpha particle; therefore, if the individual masses are added to the thorium nucleus, one would get a mass greater than the original uranium nucleus. Because of this, the spontaneous decay of four individual nucleons is not energetically feasible.

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**EK:** 1.C.4 In certain processes, mass can be converted to energy and energy can be converted to mass according to  $E = mc^2$ , the equation derived from the theory of special relativity. 4.C.4 Mass can be converted into energy and energy can be converted into mass. 5.B.11 Beyond the classical approximation, mass is actually part of the internal energy of an object or system with  $E = mc^2$ . 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

**SP:** 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.1 The student can connect phenomena and models across spatial and temporal scales. 6.2 The student can construct explanations of phenomena based on evidence produced through scientific practices.

**LO:** 1.C.4.1 The student is able to articulate the reasons that the theory of conservation of mass was replaced by the theory of conservation of mass-energy. 4.C.4.1 The student is able to apply mathematical routines to describe the relationship between mass and energy and apply this concept across domains of scale. 5.B.11.1 The student is able to apply conservation of mass and conservation of energy concepts to a natural phenomenon and use the equation  $E = mc^2$  to make a related calculation. 5.D.1.6 The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed.